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A Coup d'oeil Of The Average Blood Glucose and SGPT Levels of Drug Induced Diabetic Experimental Rats Treated with the Cissampelos Pareira L. (Menispermaceae) Root Extract

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Abstract –We discuss a three dimensional model to predict the SGPT levels of the drug induced diabetic rats belonging to the group G4T (group 4 Test) category in the experiments of Ankit Kumar et al. (see, Ankit Kumar, Ravindra Semwal, Ashutosh Chauhan, Ruchi Badoni Semwal, Subhash Chandra, Debabrata Sircar, Partha Roy and Deepak Kumar Semwal, Evaluation of antidiabetic effect of Cissampelos pareira L. (Menispermaceae) root extract in streptozotocin-nicotinamide-induced diabetic rats via targeting SGLT2 inhibition, Phytomedicine Plus 2 (2022) 100374, 11pp., https://doi.org/10.1016/j.phyplu.2022.100374). As per the experimental procedure of Ankit Kumar et al. (op. cit.), the rats of the group G4T were orally administered the Cissampelos pareira root extract at the rate of 500mg/kg/day for twenty eight days after the induction of diabetes in them using streptozotocin and nicotinamide. Treating the SGPT levels of the rats of the group G4T as a function of the number of days of the experimental study and their weekly measured average blood glucose levels, our model explains about 95.71% of the variance in the values of the SGPT of these rats based on the residual sum of squares approach. From the applications point of view, we also present our detailed analysis of the correlation matrix of the dataset under examination.

Keywords: Diabetes, Cissampelos pareira, blood glucose, regression, SGPT, correlation matrix, eigenvalues, eigenvectors.

2020 Mathematics Subject Classification 62J02, 62J05, 62J10, 62J99, 62P05, 62P20, 91B62, 91B74, 91B99, 91G70, 91G99.

1.INTRODUCTION

Towards the search of an effective herbal remedy for the treatment of diabetes mellitus, scientists are conducting a number of studies on animals. Among the many medicinal plants which have been used since antiquity by the mankind for the treatment of this ailment and its accompanying symptoms, the climber Cissampelos pareiara L. [1] belonging to the Menispermaceae family also holds a great promise in this direction, therefore, many workers have studied the different aspects of this and other plants to investigate in detail their medicinal properties, the mechanism of the therapeutic action of the phytomolecules present in these plants. Some typical works pertaining to this are [2-11]. As diabetes mellitus remains an incurable disorder till date, the effects of this disease in those people who are suffering from it are many [12-14]. The long term complications of diabetes in humans include diabetic nephropathy (which leads to chronic kidney disease), diabetic neuropathy leading to neuropathic pain, vision loss and autonomic dysfunction [13]. To understand and minimize these complications of diabetes, researchers in the field of medicine and life sciences often study the effect of diabetes and the drugs used to mange this ailment on other related organs of the body like liver, kidneys, etc.

Working on these already well defined lines of research related to the search and development of alternative remedies for the treatment of diabetes mellitus, the work of Ankit Kumar et al. [15] also focused on the effect of the apozem, which was the aqueous-ethanolic extract of the roots of Cissampelos pareira L., on the liver and kidneys of the experimental rats. In this work and some of our forthcoming papers on this topic we set our lance on the average blood glucose levels and the SGPT (Serum Glutamic Pyruvic Transaminase) levels of the diabetic experimental rats which were classified under the group 4 test (G4T) of these experiments. We are interested in examining in what manner, if any, the SGPT levels of the rats of this group depended on the number of days of this experiment and the weekly recorded average blood glucose levels of these animals after diabetes was induced in them by injecting them with streptozotocin-nicotinamide and to counter the effects of this disease they were administered with the extract of Cissampelos pareira L. for a period of four weeks beginning from day zero (basal day). By examining a number of rival mathematical models for this purpose, using the techniques of regression analysis [16-24], our efforts culminated into the development of some competing three dimensional surface models, which emerged as the best models based on our experience, which successfully explain the pattern of the observed values of the SGPT levels of the rats of the group G4T of the work of Ankit Kumar at al. [15] as functions of the number of days of the experiment and their measured average blood glucose levels. We aim to discuss these models one by one in our upcoming papers. This paper discusses the first model proposed by us in this direction as part of our already ongoing studies [25-28].

The structure of the paper consists of the following. The secondary data for this work is narrated in the second section of the paper. The core model developed by us is thoroughly discussed in the third section with its complete details. In section four we give many mathematical features of the correlation matrix of the model developed in section three. The conclusions in section five finish the paper.

1.1 Abbreviations Used in the Paper

For the sake of convenience, we use the following abbreviations at different places in our work. Some of these abbreviations, like SGPT, are also used in medical terminology and for the sake of conformity we have also used the very same abbreviations as are used by the authors of [15].

G4T: group 4 Test
Day: Day of Experiment
SGPT: Serum Glutamic Pyruvic Transaminase
1.2345E-03 =1.2345×10^{-3}
BG: Blood Glucose
G4BGm: Mean of the blood glucose levels of the G4T rats
1.2345E+03 =1.2345×10^{3}
G4SGPT: SGPT levels of the group G4T rats

2. DATA FOR THE STUDY

We reproduce below in Table 1 the secondary data for our study, which we have gratefully taken from Table 3 and Table 4 of the work of Ankit Kumar et al. (see, [15, Table 3 and Table 4, p. 6]). We place on record our sincere thanks to the learned authors of [15] and the Publisher of the Journal [15] which is the original source of the data given in Table 1 below.

Template sample paragraph. Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

Table -1: Measurements of the average blood glucose levels in mg/dL and the SGPT levels in (U/L) of the experimental rats showing the effect of the Cissampelos Pareira Root Extract on it.

S. No.	Day	G4BGm (mg/dL)	G4SGPT (U/L)
1.	0	303.9	36.25
2.	7	288.54	37.17
3.	14	250.24	37.98
4.	21	225.35	38.64
5.	28	198.56	60.39

Source: A. Kumar et al. [15]

3. A SURFACE FIT TO MEASUREMENTS OF BLOOD GLUCOSE LEVELS AND SGPT LEVELS OF THE RATS OF THE GROUP G4T

In this section we discuss in detail the three dimensional model formulated by us to explain the variation in the values of the G4SGPT levels of the diabetic rats as reproduced in Table 1 above by assuming that it depends on the values of the day and the corresponding measured average values of the blood glucose levels of these experimental animals. Our detailed studies of a number of possible mathematical models led us to conclude that one of the interesting models which can explain with a great accuracy the trend of the values of G4SGPT in Table 1 with the corresponding values of the Day and G4BGm is the following equation:

$$y = a + bx_1 + \frac{c}{x_2} + \frac{d}{x_2^2} \quad (3.1)$$

where, y represents the response variable G4SGPT (in U/L), the predictor x_1 represents the variable Day and the predictor x_2 represents the variable G4BGm (in mg/dL). We point out that the response y and the predictors x_1 and x_2 of (3.1) are respectively mentioned as 'y', 'x_1' and 'x_2' in the tables and graphs below.

From Table 2 we note certain important points about the model of (3.1). The first thing to be noted is that the coefficient of multiple determination for the model is 0.9570706653, which shows that about 95.71% of variation in the values of G4SGPT (response) is explained by the model with the change in values of the predictors- Day and G4BGm. Further, since the adjusted coefficient of multiple determination is 0.8282826614, as there are four parameters in this model, therefore, according to this criterion only 82.83% of the variability of G4SGPT can be explained by the model, since the adjusted coefficient of multiple determination is always smaller than the coefficient of multiple determination. The value of the Durbin-Watson statistic is 3.12407408002259. We also observe that the highest P-Values of the t - statistic for the four regression parameters, a, b, c, d is 0.77884, which corresponds to the parameter b. This shows that there are about 77.88% chances that the actual value of the parameter b may be zero, which guides us to the fact that we can remove the parameter b from our model without disturbing much the accuracy of our proposed regression model of (3.1). A number of our numerical experiments conducted so far by us on this issue have definitely shown us that this is indeed true. But we have decided at present to restrain ourselves from moving into that direction, and we propose to expound on this issue in our coming manuscripts. Yet, since there are only two predictors Day (x1) and G4BGm (x2), we do not remove this parameter from our model of (3.1) at present, because removing the parameter b from our model of (3.1) at present means that the equation (3.1) reduces to the case of a single variable x_2, thus we would study the variation of the response y = G4SGPT with only a single predictor x_2 = G4BGm, which we have actually done and the results of these studies are earmarked for our future communications on this subject.

Besides showing the values of the four regression parameters a,b,c,d of (3.1) with their respective standard errors, Table 2 also displays the 68%, 90%, 95% and the 99% confidence intervals of these parameters. The lowermost portion of the table shows the Variance Analysis of the model (3.1), with a P-Value of 0.26191, showing that there is a 73.81% chance that at least one of the four regression parameters of (3.1) actually has a nonzero value, which leads us to infer that our model of (3.1) is reliable. It is also observed from Table 2 that the numerical magnitude of the t - statistic is the greatest for the parameter d at 2.05, this underscores the fact that the parameter d is the most significant parameter of the model (3.1).

Table -2: Details of the Model of (3.1) for the dataset of Table 1.

Model Definition: Y = a+b*x1+c/x2+d/x2^2
Number of observations = 5
Number of missing observations = 0
Solver type: Nonlinear
Nonlinear iteration limit = 250
Diverging nonlinear iteration limit =10
Number of nonlinear iterations performed = 4
Residual tolerance = 0.0000000001
Sum of Residuals = 3.00630631500098E-11
Average Residual = 6.01261263000197E-12
Residual Sum of Squares (Absolute) = 18.11602983058

Residual Sum of Squares (Relative) = 18.11602983058
Standard Error of the Estimate = 4.25629296813318
Coefficient of Multiple Determination (R^2) = 0.9570706653
Proportion of Variance Explained = 95.70706653%
Adjusted coefficient of multiple determination (Ra^2) = 0.8282826614
Durbin-Watson statistic = 3.12407408002259
Regression Variable Results
VariableValue Standard Error t-ratio Prob(t)
a 309.613245242293204.55882574672 1.513565812 0.37169
b 0.6101736693415081.68516498370101 0.3620854191 0.77884
c -139058.22919107789004.3364950211 -1.562375887 0.36246
d 17078569.61191578329445.35498787 2.050384976 0.28888
68% Confidence Intervals
VariableValue 68% (+/-) Lower Limit Upper Limit
a 309.613245242293372.092504033284-62.4792587909907681.705749275578
b 0.6101736693415083.06531510535214-2.455141436010633.67548877469364
c -139058.229191077161898.888084443-300957.11727552122840.6588933662
d 17078569.611915715151261.10072291927308.5111927332229830.7126386
90% Confidence Intervals
VariableValue 90% (+/-) Lower Limit Upper Limit
a 309.6132452422931291.54351399964-981.9302687573491601.15675924194
b 0.61017366934150810.6397946740914-10.029621004749911.2499683434329

c -139058.229191077561955.579762264-701013.808953341422897.350571187
d 17078569.611915752590452.0823224-35511882.470406769669021.694238
95% Confidence Intervals
VariableValue 95% (+/-) Lower Limit Upper Limit
a 309.613245242293 2599.16535170298 -2289.55210646068 2908.77859694527
b 0.61017366934150 21.4120433159018 -20.8018696465603 22.0222169852433
c -139058.229191077 1130906.90037304 -1269965.12956411 991848.67118196
d 17078569.6119157 105835598.569547 -88757028.9576312 122914168.181462
99% Confidence Intervals
VariableValue 99% (+/-) Lower Limit Upper Limit
a 309.61324524229313021.7034499718-12712.0902047296 13331.3166952141
b 0.6101736693415081 07.273389949947-106.663216280606 107.883563619289
c -139058.2291910775665793.55043181-5804851.77962288 5526735.32124073
d 17078569.6119157530231667.68514-513153098.073224 547310237.297056
Variance Analysis
Source DFSum of Squares Mean Square F Ratio Prob(F)
Regression3403.88049016942134.6268300564737.4313650020.26191
Error 118.1160298305818.11602983058
Total 4421.99652

Table 3 depicts the computations made from the model of (3.1) for the values of the sample of Table 1, from where see that the residuals vary from a minimum of -3.0175733 U/L to a maximum of 2.790554016 U/L and the largest percentage error is 7.81% for the G4SGPT value of 38.64 U/L, which is quite manageable. Table 4 below gives some brief descriptive statistics of the sample of Table 1, which we have included here only to give the reader a glimpse of the salient features of the dataset being studied by us. From the correlation matrix of the sample of Table 1 included in Table 4, it is evident that the response G4SGPT is strongly positively correlated ($r = 0.766$) with the predictor Day, as we can see from Table 1 that G4SGPT increases with increase in Day. On the contrary, the response SGPT is strongly negatively correlated ($r = -0.761$) with the predictor G4BGm, as G4SGPT levels increase with decreasing levels of G4BGm. The two predictors Day and G4BGm are most strongly negatively correlated with each other ($r = -0.994$), which is also very much clear from Table 1, where the G4BGm levels of the rats of the Test Group 4 of the authors of [15] show a decline with increasing number of the predictor Day. This fact also supports to some extent the experimental finding of Ankit Kumar et al. [15] that the ethanolic extract of the roots of the plant Cissampelos pareira promises an effective herbal remedy for the lowering of the elevated blood glucose levels of the diabetic experimental rats! In section 4 we discuss the various characteristics of this correlation matrix of the model of (3.1) which is displayed in Table 4. The summary report of the whole computational procedure performed by us for this model is shown in Table 5, which shows that the computations successfully converged in four iterations, with the final merit function value (Residual Sum of Squares) as 18.11602983058. In Table 6 we tabulate the results of our evaluations of the response G4SGPT (y) for different values of the predictors Day (x1) and G4BGm (x2) based on the model of (3.1) on an approximately daily basis for a period of one month and the partial derivatives of the response with respect to these two predictors and the parameters are also calculated at these points.

Table -3: Evaluation by the Model of (3.1) for the dataset of Table 1.

x1 Value	x2 Value	y Value	Calc y	Residual	% Error	Abs Residual	Min Residual	Max Residual
0	303.9	36.25	36.95698428	-0.7069842771	-1.950301454	0.7069842771	-3.0175733	2.790554016
7	288.54	37.17	37.08186587	0.08813412531	0.2371109102	0.08813412531		
14	250.24	37.98	35.18944598	2.790554016	7.347430269	2.790554016		
21	225.35	38.64	41.6575733	-3.0175733	-7.80945471	3.0175733		
28	198.56	60.39	59.54413056	0.8458694354	1.400677985	0.8458694354		

The plot of the surface generated by the model of (3.1) corresponding to the dataset of Table 1 is shown in Fig. 1, while the Residual Error Plot of this model corresponding to Table 3 is displayed in Fig. 2. Fig. 3 gives the Residual Normal Probability Plot of the model of (3.1) from which we see that the Residual Normal Probability Plot of the model shows that the residuals deviate both above and below the standard reference line, yet we conclude that the residuals of the model are normally distributed. This fact is confirmed from the brief statistics of the Residuals presented in Table 7, where both the standardized skewness and the standardized kurtosis lie well within the range of -2 to +2, which shows that these Residuals of the Model of (3.1) come from a Normal Distribution. We further confirm this fact from Fig. 4 which depicts the Normal Probability of the Residuals of the Model of (3.1) using the mean and sigma method with their 95% confidence limits (the pink colored curves) around the reference line (the blue colored line). The reader may note from this figure that the five Residuals lie very close to the reference line, which signifies the fact that the Residuals are drawn from a normal population. Another authenticity of this fact comes from the displayed high P-Value =0.9799 of the Shapiro – Wilk test performed by us in

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this case, which indicates that these residuals come from a normal distribution with a very high probability. A Surface Plot of the dataset of Table 1 is shown by us in Fig. 5.

Table -4: Descriptive Statistics and Correlation Matrix of the sample of Table 1.

Variable	*Day*	*G4BGm (mg / dL)*	*SGPT (U / L)*
Number of Points	5	5	5
Missing Points	0	0	0
Maximum Value	28	303.9	60.39
Minimum Value	0	198.56	36.25
Range	28	105.34	24.14
Average	14	253.318	42.086
Standard Deviation	11.06797181	43.55814987	10.27127694
Correlation Matrix			
	Day	*G4BGm (mg / dL)*	*SGPT (U / L)*
Day	1	−0.994134261	0.765841066
G4BGm (mg / dL)	−0.994134261	1	−0.760856601
SGPT (U / L)	0.765841066	−0.760856601	1

Table – 5: Summary Report of the procedure performed.

Beginning non-linear solution for model a+b*x1+c/x2+d/x2^2
Obtaining initial estimates
Initial estimates successful
Solving with 1 initial condition(s)
Beginning non-linear iterations
Solution converged to residual change less than 0.000000001%
Final Merit Function = 18.11602983058
Total number of non-linear iterations = 4
Residual Sum of Squares = 18.11602983058
Solution Complete

Table – 6: Evaluations of SGPT levels for different values of the Day and G4BGm from the model of (3.1) for the sample of Table 1.

S. No.	x1= Day	x2= G4BGm	y= G4SGPT	$\partial y / \partial x_1$	$\partial y / \partial x_2$	$\partial y / \partial a$	$\partial y / \partial b$	$\partial y / \partial c$	$\partial y / \partial d$

		(mg /dL)	(U/L)						
1.	0	305	37.27581	6.101736627684E-01	2.909713386972E-01	1.000000000000E+00	-1.879591836735E-06	3.278694687198E-03	1.074983885186E-05
2.	0.612245	291.7755	34.00418	6.101736704701E-01	2.583220957305E-01	1.000000000000E+00	6.122438492293E-01	3.427299000757E-03	1.174637844059E-05
3.	1	301.4	36.85216	6.101736708048E-01	2.832387553396E-01	1.000000000000E+00	9.999986574344E-01	3.317856269394E-03	1.100817022436E-05
4.	1.22449	296.1837	35.54397	6.101736704701E-01	2.705513603689E-01	1.000000000000E+00	1.224487698459E+00	3.376287562642E-03	1.139931770565E-05
5.	2	283.4	32.79864	6.101736667866E-01	2.307395887491E-01	1.000000000000E+00	1.999996240816E+00	3.528588142538E-03	1.245093427966E-05
6.	2.44898	274.1429	31.10695	6.101736704701E-01	1.924322679736E-01	1.000000000000E+00	2.448975396917E+00	3.647739480378E-03	1.330600331671E-05
7.	3	290.6	35.15952	6.101736681260E-01	2.548086996843E-01	1.000000000000E+00	2.999994361224E+00	3.441162696474E-03	1.184160070360E-05
8.	3.673469	302.7959	38.881	6.101736719198E-01	2.863315573043E-01	1.000000000000E+00	3.673462095378E+00	3.302560832546E-03	1.090690805266E-05
9.	4	283.4	34.01899	6.101736687957E-01	2.307395887491E-01	1.000000000000E+00	3.999992481633E+00	3.528588142538E-03	1.245093427966E-05

10.	4.285714	293.9796	36.822	6.101736699425E-01	2.646188566626E-01	1.000000000000E+00	4.285705944607E+00	3.401601140240E-03	1.157089031728E-05
11.	5	204.2	41.25497	6.101736691975E-01	-6.766506374703E-01	1.000000000000E+00	4.999990602041E+00	4.897168852083E-03	2.398226276581E-05
12.	5.510204	285.1633	35.35295	6.101736688930E-01	2.370597500477E-01	1.000000000000E+00	5.510196602190E+00	3.506769207662E-03	1.229743027581E-05
13.	6	218.6	34.54055	6.101736694654E-01	-3.598512747274E-01	1.000000000000E+00	5.999988722449E+00	4.574574014617E-03	2.092672741521E-05
14.	6.734694	296.1837	38.90615	6.101736694184E-01	2.705513603689E-01	1.000000000000E+00	6.734681341524E+00	3.376287562642E-03	1.139931770565E-05
15.	7	243.8	30.83823	6.101736691975E-01	-1.757927918874E-02	1.000000000000E+00	6.999986842857E+00	4.101730433123E-03	1.682419254601E-05
16.	7.346939	223.449	33.82367	6.101736701624E-01	-2.764904125076E-01	1.000000000000E+00	7.346925190753E+00	4.475302550449E-03	2.002833291806E-05
17.	8	236.6	31.84491	6.101736697488E-01	-9.482707314189E-02	1.000000000000E+00	7.999984963265E+00	4.226548362500E-03	1.786371106055E-05
18.	8.571429	258.7143	32.50488	6.101736694563E-01	1.050563471030E-01	1.000000000000E+00	8.571417492294E+00	3.865274859547E-03	1.494034973985E-05

19.	9	254.6	32.3938	6.101736690189E-01	7.556508592288E-02	1.000000000000E+00	8.999983083673E+00	3.927737154734E-03	1.542711915668E-05
20.	9.795918	291.7755	39.60781	6.101736693522E-01	2.583220957305E-01	1.000000000000E+00	9.795892222741E+00	3.427299000757E-03	1.174637844059E-05
21.	10	279.8	36.87389	6.101736691975E-01	2.169073130514E-01	1.000000000000E+00	9.999981204082E+00	3.573988132936E-03	1.277339117436E-05
22.	10.40816	221.2449	36.34098	6.101736685685E-01	-3.131378363837E-01	1.000000000000E+00	1.040814043691E+01	4.519886693864E-03	2.042937572537E-05
23.	11	272.6	36.03326	6.101736713483E-01	1.851267760086E-01	1.000000000000E+00	1.099997932449E+01	3.668388229771E-03	1.345707220432E-05
24.	11.63265	282.9592	38.57488	6.101736682160E-01	2.291140028067E-01	1.000000000000E+00	1.163262813537E+01	3.534085053942E-03	1.248975716850E-05
25.	12	283.4	38.90038	6.101736683452E-01	2.307395887491E-01	1.000000000000E+00	1.199998388921E+01	3.528588142538E-03	1.245093427966E-05
26.	12.2449	201.4082	47.6696	6.101736681810E-01	-7.526975098709E-01	1.000000000000E+00	1.224487698459E+01	4.965050477564E-03	2.465172624476E-05
27.	13	229.4	35.9005	6.101736689502E-01	-1.869761578406E-01	1.000000000000E+00	1.299997556531E+01	4.359203760102E-03	1.900265742209E-05

28.	13.46939	221.2449	38.20886	6.101736690134E-01	-3.131378363837E-01	1.000000000000E+00	1.346936468304E+01	4.519886693864E-03	2.042937572537E-05
29.	14	222.2	38.24128	6.101736691975E-01	-2.970108578943E-01	1.000000000000E+00	1.399997368571E+01	4.500458504030E-03	2.025412674649E-05
30.	14.69388	230.0612	36.81346	6.101736691348E-01	-1.778177564280E-01	1.000000000000E+00	1.469385238150E+01	4.346677664879E-03	1.889360672236E-05
31.	15	243.8	35.71962	6.101736701319E-01	-1.757927918874E-02	1.000000000000E+00	1.499997180612E+01	4.101730433123E-03	1.682419254601E-05
32.	15.30612	210.2245	43.92021	6.101736691084E-01	-5.299573739041E-01	1.000000000000E+00	1.530609945053E+01	4.756828436245E-03	2.262741677187E-05
33.	16	240.2	36.45856	6.101736692979E-01	-5.450429209484E-02	1.000000000000E+00	1.599996992653E+01	4.163205160680E-03	1.733227720991E-05
34.	16.53061224	298.2653061	45.45210178	6.101736691116E-01	2.758352861196E-01	1.000000000000E+00	1.653058116920E+01	3.352723023847E-03	1.124075167464E-05
35.	17	272.6	39.6943	6.101736689139E-01	1.851267760086E-01	1.000000000000E+00	1.699997717638E+01	3.668388229771E-03	1.345707220432E-05
36.	17.7551	296.1837	45.63051	6.101736694459E-01	2.705513603689E-01	1.000000000000E+00	1.775507616261E+01	3.376287562642E-03	1.139931770565E-05

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37.	18	276.2	41.00129	6.101736690189E-01	2.017408725773E-01	1.000000000000E+00	1.799996616735E+01	3.620571613307E-03	1.310853880708E-05
38.	18.36735	216.8367	42.75008	6.101736693816E-01	-3.927522325553E-01	1.000000000000E+00	1.836731547688E+01	4.611774112018E-03	2.126846046028E-05
39.	19	211.4	45.56656	6.101736691129E-01	-5.038635581963E-01	1.000000000000E+00	1.899996428776E+01	4.730377859959E-03	2.237647469799E-05
40.	19.59184	221.2449	41.94461	6.101736695316E-01	-3.131378363837E-01	1.000000000000E+00	1.959180317534E+01	4.519886693864E-03	2.042937572537E-05
41.	20	279.8	42.97562	6.101736698983E-01	2.169073130514E-01	1.000000000000E+00	1.999996240816E+01	3.573988132936E-03	1.277339117436E-05
42.	20.20408	203.6122	50.93437	6.101736701269E-01	-6.922108695123E-01	1.000000000000E+00	2.020404202458E+01	4.911303657479E-03	2.412090361597E-05
43.	21	297.8	48.05112	6.101736691975E-01	2.746802941088E-01	1.000000000000E+00	2.099996052857E+01	3.357964672919E-03	1.127592674457E-05
44.	21.42857	302.7959	49.71469	6.101736698465E-01	2.863315573043E-01	1.000000000000E+00	2.142852972303E+01	3.302560832546E-03	1.090690805266E-05
45.	22	272.6	42.74517	6.101736694171E-01	1.851267760086E-01	1.000000000000E+00	2.199997046356E+01	3.668388229771E-03	1.345707220432E-05

46.	22.65306	274.1429	43.43495	6.101736693325E-01	1.924322679736E-01	1.000000000000E+00	2.265301742149E+01	3.647739480378E-03	1.330600331671E-05
47.	23	265.4	42.15542	6.101736690577E-01	1.470477976628E-01	1.000000000000E+00	2.299995676939E+01	3.767902571844E-03	1.419708979091E-05
48.	23.26531	203.6122	52.80224	6.101736695416E-01	-6.922108695123E-01	1.000000000000E+00	2.326524877900E+01	4.911303657479E-03	2.412090361597E-05
49.	24	211.4	48.61743	6.101736692800E-01	-5.038635581963E-01	1.000000000000E+00	2.399995488980E+01	4.730377859959E-03	2.237647469799E-05
50.	24.4898	210.2245	49.52384	6.101736690814E-01	-5.299573739041E-01	1.000000000000E+00	2.448975396917E+01	4.756828436245E-03	2.262741677187E-05
51.	25	236.6	42.21786	6.101736688760E-01	-9.482707314189E-02	1.000000000000E+00	2.499995301020E+01	4.226548362500E-03	1.786371106055E-05
52.	25.71429	258.7143	42.965	6.101736692100E-01	1.050563471030E-01	1.000000000000E+00	2.571422233468E+01	3.865274859547E-03	1.494034973985E-05
53.	26	229.4	43.83276	6.101736690313E-01	-1.869761578406E-01	1.000000000000E+00	2.599996509329E+01	4.359203760102E-03	1.900265742209E-05
54.	26.32653	197	59.86491	6.101736696744E-01	-8.845439029879E-01	1.000000000000E+00	2.632649465491E+01	5.076151673073E-03	2.576731580804E-05

55.	27	200.6	57.29036	6.101736690189E-01	-7.757545486488E-01	1.000000000000E+00	2.699994925102E+01	4.985054235271E-03	2.485076572859E-05
56.	27.55102	214.6327	49.26657	6.101736694764E-01	-4.359731399539E-01	1.000000000000E+00	2.755096821533E+01	4.659131062487E-03	2.170750225743E-05
57.	28	243.8	43.65187	6.101736691975E-01	-1.757927918874E-02	1.000000000000E+00	2.799994737143E+01	4.101730433123E-03	1.682419254601E-05
58.	28.16327	201.4082	57.38257	6.101736694338E-01	-7.526975098709E-01	1.000000000000E+00	2.816321706455E+01	4.965050477564E-03	2.465172624476E-05
59.	29	218.6	48.57454	6.101736689616E-01	-3.598512747274E-01	1.000000000000E+00	2.899994549184E+01	4.574574014617E-03	2.092672741521E-05
60.	29.38776	267.5306	46.37924	6.101736695176E-01	1.590355912703E-01	1.000000000000E+00	2.938770476301E+01	3.737897196042E-03	1.397187544818E-05
61.	30	204.2	56.50931	6.101736694915E-01	-6.766506374703E-01	1.000000000000E+00	2.999994361224E+01	4.897168852083E-03	2.398226276581E-05
62.	30.35897436	283.6111111	50.15133737	6.101736694955E-01	2.315115865166E-01	1.000000000000E+00	3.035891729752E+01	3.525961573638E-03	1.243240501877E-05

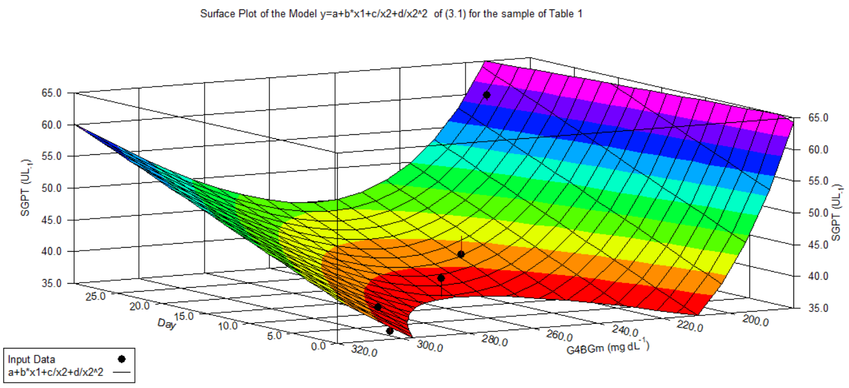

Fig -1: Plot of the surface of the Model of (3.1) for the sample of Table 1.

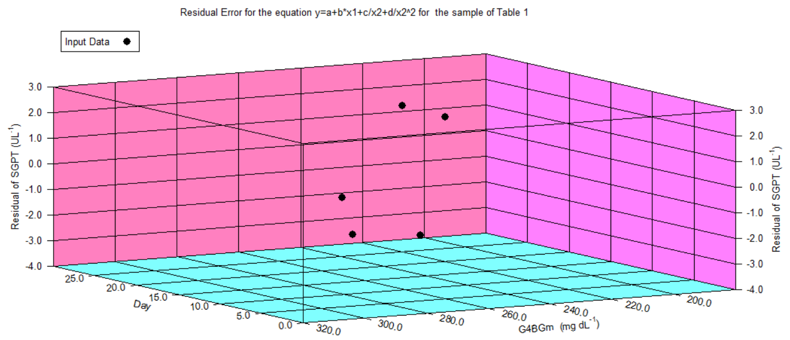

Fig -2: Residual Error Plot of the Model of (3.1) for the sample of Table 1.

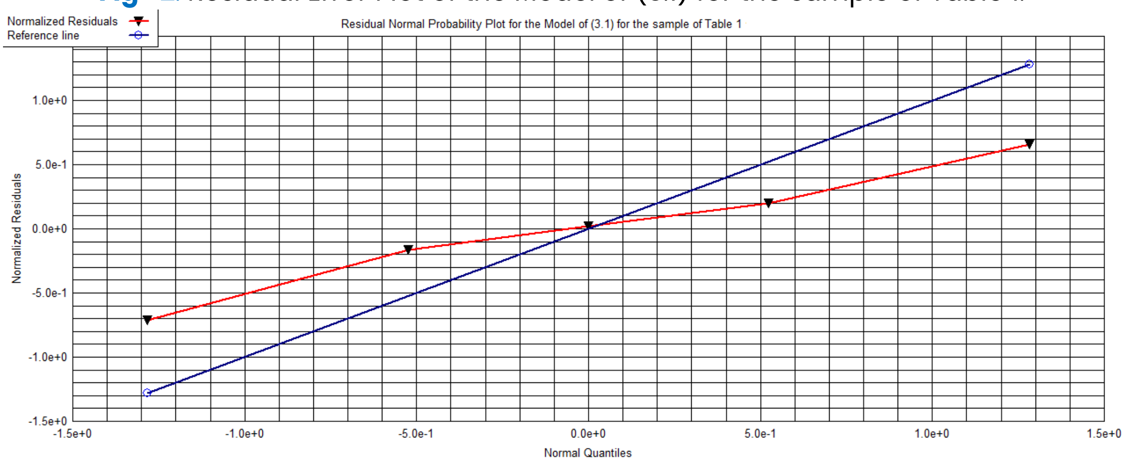

Fig -3: Residual Normal Probability Plot of the Model of (3.1) for the sample of Table 1.

Table – 7: Brief Summary Statistics of the Residuals of the Model of (3.1)

Summary Statistics for Residuals	
Count	5
Average	-1.8E-7
Median	0.0881341
Standard deviation	2.12814
Coeff. of variation	-1.1823E9%
Minimum	-3.01757
Maximum	2.79055
Range	5.80812
Stnd. Skewness	-0.216818
Stnd. Kurtosis	0.362728

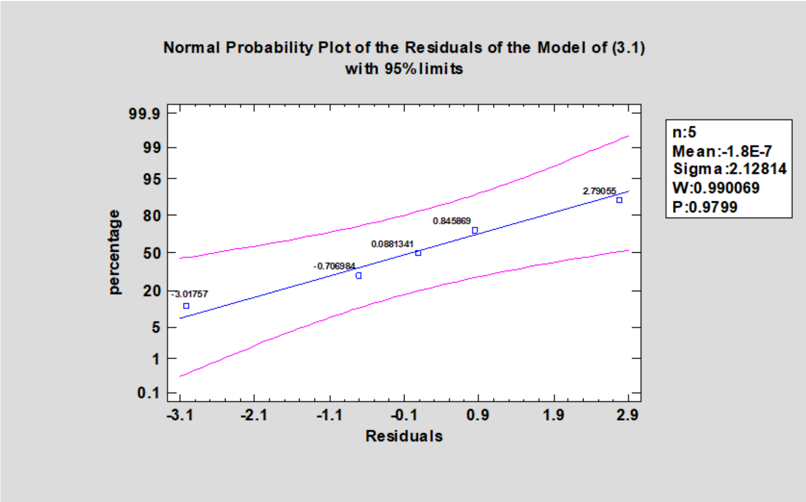

Fig -4: Normal Probability Plot of the Residuals of the Model of (3.1).

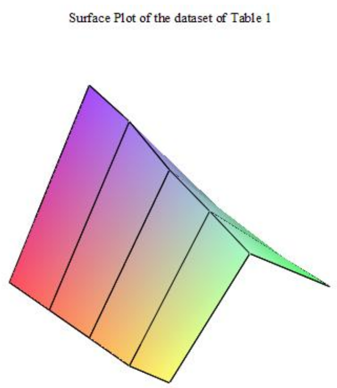

Fig -5: A Surface Plot of the dataset of Table 1.

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4. CHARACTERISTICS OF THE CORRELATION MATRIX OF THE MODEL OF (3.1)

This section deals with some interesting properties of the correlation matrix of the model defined by (3.1) for the sample of Table 1. We have used the various concepts of Linear Algebra and Matrix Theory, with which the mathematicians are mostly aware of and for the sake of our other interested readers we mention some of the handpicked references [29-34] and for the other concepts utilized by us for our exposition of this section they may access the easily available resources.

The correlation matrix of the model of (3.1) displayed in Table 4 above is essentially a symmetric matrix of order three, which we reproduce below in a convenient mathematical form for the purpose of our study in this section.

$$A := \begin{bmatrix} 1 & -0.994134261 & 0.765841066 \\ -0.994134261 & 1 & -0.760856601 \\ 0.765841066 & -0.760856601 & 1 \end{bmatrix} \quad (4.1)$$

It is a real symmetric positive definite matrix with rank three, which obviously has an empty null space and the value of its determinant and the spanning vectors for its row and column spaces are as shown below:

$$\begin{aligned} &\text{Rank(A)} = 3; \quad \text{RowDimension(A)}=3; \quad \text{ColumnDimension(A)} = 3; \\ &\text{Determinant(A)} = 0.004836350002; \\ &\text{Determinant(A, method = algnum)} = 0.004836349937 \\ &\text{NullSpace(A)} = \{\}; \text{RowDimension(A)=3=ColumnDimension(A)} \\ &\text{RowSpace(A)} = [[1. \quad 0. \quad 0.],[0. \quad 1. \quad 0.],[0. \quad 0. \quad 1.]] \\ &\text{ColumnSpace(A)} = \left[\begin{bmatrix}1.\\0.\\0.\end{bmatrix},\begin{bmatrix}0.\\1.\\0.\end{bmatrix},\begin{bmatrix}0.\\0.\\1.\end{bmatrix}\right]; \text{ IsDefinite(A, query = 'positive\_definite') = true} \end{aligned} \quad (4.2)$$

Invoking the Gram –Schmidt orthogonalization process, we can have an *orthogonal basis* for A as

$$\text{ord} := \left[\begin{bmatrix} 1 \\ -0.994134261 \\ 0.765841066 \end{bmatrix}, \begin{bmatrix} 0.00436982009999998 \\ 0.00735288320000005 \\ 0.00383882889999998 \end{bmatrix}, \begin{bmatrix} -0.201889183079787 \\ -0.0105190338788632 \\ 0.249962900348282 \end{bmatrix}\right], \quad (4.3)$$

and the corresponding *orthonormal basis* for A is given by

$$\text{orthnor} := \left\{\begin{bmatrix} 0.6231992836 \\ -0.6195437593 \\ 0.4772716037 \end{bmatrix}, \begin{bmatrix} 0.466097700949604 \\ 0.784279873414241 \\ 0.409460637665358 \end{bmatrix}, \begin{bmatrix} -0.627993265891283 \\ -0.0327203386473544 \\ 0.777530602416366 \end{bmatrix}\right\} \quad (4.4)$$

The characteristic polynomial of A as a function of a variable λ is given by

$$\text{CharacteristicPolynomial}(A, \lambda) = -0.0048363500 + 0.846281766\lambda - 3\lambda^2 + \lambda^3 \quad (4.5)$$

which on solving for λ yields the following eigenvalues and the corresponding eigenvectors for A

$$v,\ e := \text{Eigenvectors(A)} \Rightarrow$$
$$v := \begin{bmatrix} 2.68554591477729+0.I \\ 0.00583529437354411+0.I \\ 0.308618790849168+0.I \end{bmatrix},$$
$$e := \begin{bmatrix} -0.595950234052158+0.I & 0.710388726476149+0.I & -0.374421120970457+0.I \\ 0.594947020166279+0.I & 0.703756528606398+0.I & 0.388284421061595+0.I \\ -0.539334183719560+0.I & -0.00863746140177329+0.I & 0.842047523915420+0.I \end{bmatrix}. \quad (4.6)$$

The scheme of (4.6) entails that corresponding to the first eigenvalue $\lambda_1 = 2.68554591477729 + 0.I$ of A in the vector v, the corresponding eigenvector v_1 is given by the first column of the matrix e as

$$v_1 = \begin{bmatrix} -0.595950234052158+0.I \\ 0.594947020166279+0.I \\ -0.539334183719560+0.I \end{bmatrix};\ I = \sqrt{-1}$$

and similarly for the other two remaining eigenvectors v_2, v_3 of the matrix A corresponding to its two eigenvalues λ_2, λ_3. The inverse of the matrix A is given by

$$\text{MatrixInverse(A)} =$$
$$A^{-1} = \begin{bmatrix} 87.0692232885649 & 85.0722209582725 & -1.95342590127446 \\ 85.0722209582725 & 85.4957699232816 & -0.101779481964882 \\ -1.95342590127446 & -0.101779481964882 & 2.41857418388470 \end{bmatrix}. \quad (4.7)$$

The LU Decomposition of A, which is a positive definite matrix, by Cholesky's method (Cholesky Decomposition) gives the following square lower triangular factor (matrix) L

$$\text{LUDecomposition}(A, \text{method} = \text{'Cholesky'})$$
$$= L := \begin{bmatrix} 1. & 0. & 0. \\ -0.994134261000000 & 0.108152998599132 & 0. \\ 0.765841066000000 & 0.00455134113467118 & 0.643013799946673 \end{bmatrix}, \quad (4.8)$$

and since A is a real symmetric matrix also (as we have already remarked earlier in this section), therefore, the other factor U (the square upper triangular factor (matrix)) is simply the transpose of L, i.e., $U = L^T$, such that $A = LU = LL^T$, which we verify further as under

$$\text{LUDecomposition}(A, \text{ method} = \text{'Cholesky'}, \text{ output} = [\text{'}U\text{'}])$$
$$= U := \begin{bmatrix} 1. & -0.994134261000000 & 0.765841066000000 \\ 0. & 0.108152998599132 & 0.00455134113467118 \\ 0. & 0. & 0.643013799946673 \end{bmatrix},$$
$$\text{Multiply}(L,\ U) \tag{4.9}$$
$$= \begin{bmatrix} 1. & -0.994134261000000 & 0.765841066000000 \\ -0.994134261000000 & 1. & -0.760856601000000 \\ 0.765841066000000 & -0.760856601000000 & 1.00000000000000 \end{bmatrix} = A.$$

It is instructive to have a look at the matrix plot of the correlation matrix A of the dataset of Table 1 given by (4.1). Accordingly, we draw the matrix plot of A in Fig. 6 below:

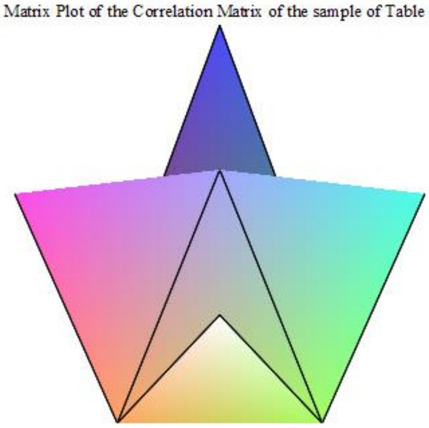

Fig -6: Matrix Plot of the Correlation Matrix of the sample of Table 1.

(4.7) above gives the inverse A^{-1} of the matrix A defined by (4.1). The matrix plot of this matrix A^{-1} is pictured in Fig.7 underneath. The Eigen Plot of the three eigenvectors v_1, v_2, v_3 given by (4.6) of the matrix A is drawn in Fig. 8. Corresponding to the system of linear equations $Ax_1 = v_1$, for the first eigenvector v_1 of the correlation matrix A, the unique solution is given by the vector $x_1 = (-.2303,\ .2133,\ -.2006)^T$ as shown in Table 8 along with the solutions for the remaining two similar systems of linear equations. The solutions of these three systems of linear equations $Ax_i = v_i, i = 1,2,3$ are shown in the Figs. (9a), 9(b) and 9(c), where a circle is plotted at the solution, since each of these three systems of linear equations have unique solutions in the three dimensional Euclidean space $\mathbb{R}^3(\mathbb{R})$. At first glance the three graphs of Figs. 9(a), 9(b) and 9(c) appear to be identical, but a closer look reveals that the markings on the three axes are different in all of them, which in fact correspond to the unique solutions x_1, x_2, x_3 of these three systems as shown in Table 8. We have also shown in Table 8 that the sum of the eigenvectors v_1, v_2, v_3 of A is the vector $(-.2600,\ 1.687,\ .2941)^T$, which is represented by the black colored vector in Fig. 9(d). By the linear transformation (operator) in $\mathbb{R}^3(\mathbb{R})$, which is defined by the equation

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$T(\vec{x}) = A\vec{x},\ \vec{x} \in \mathbb{R}^3(\mathbb{R})$, the unit sphere is transformed into the 3-space which is shown in Fig. 9(e).

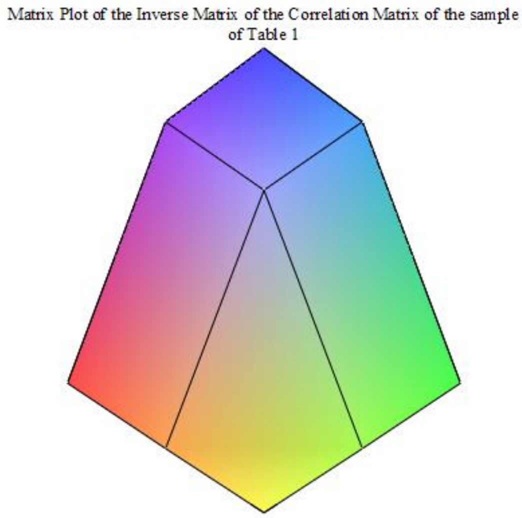

Fig -7: Matrix Plot of the Inverse Matrix of the Correlation Matrix of the sample of Table 1.

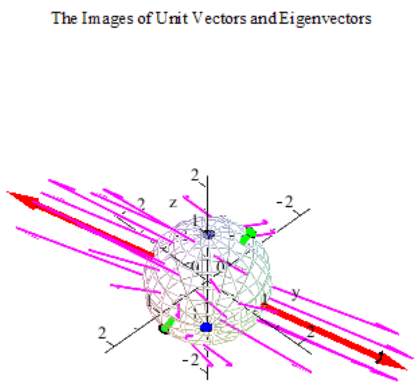

Fig -8: The Eigen Plot of the Correlation Matrix of the sample of Table 1.

Table – 8: Solutions of the systems of linear equations $Ax_i = v_i, i = 1,2,3$ and sum of the eigenvectors $v_i, i = 1,2,3$.

The solution is the point $x_1 = (-.2303,\ .2133,\ -.2006)^T$ for the system of linear equations $Ax_1 = v_1$.
The solution is the point $x_2 = (121.7,\ 120.6,\ -1.480)^T$ for the system of linear equations $Ax_2 = v_2$.
The solution is the point $x_3 = (-1.210,\ 1.261,\ 2.728)^T$ for the system of linear equations $Ax_3 = v_3$.
The sum of the three eigenvectors v_1, v_2, v_3 of the matrix A is the vector $(-.2600,\ 1.687,\ .2941)^T$.

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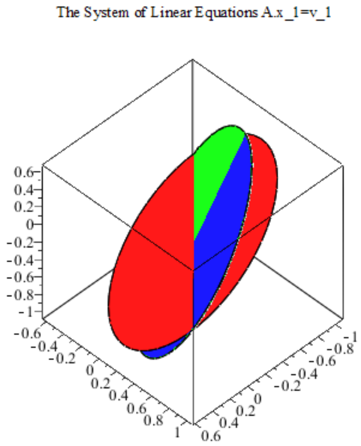

Fig -9(a): The solution of the system of linear equations $Ax_1 = v_1$.

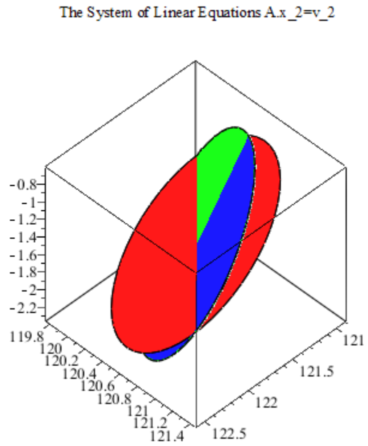

Fig -9(b): The solution of the system of linear equations $Ax_2 = v_2$.

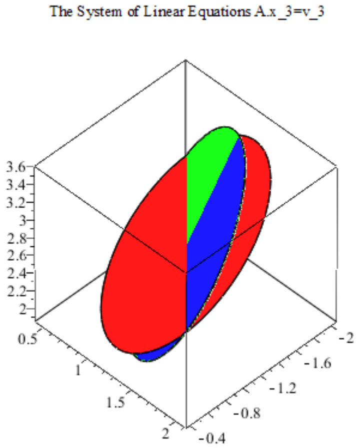

Fig -9(c): The solution of the system of linear equations $Ax_3 = v_3$.

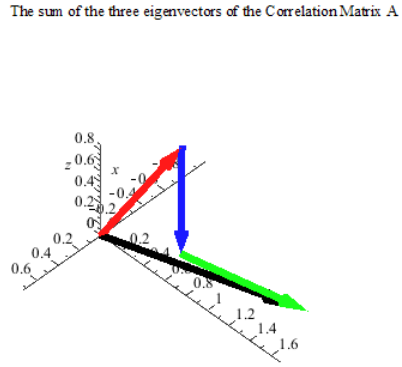

Fig -9(d): The sum of the three eigenvectors x_1, x_2, x_3 of the correlation matrix A is shown by the black colored vector.

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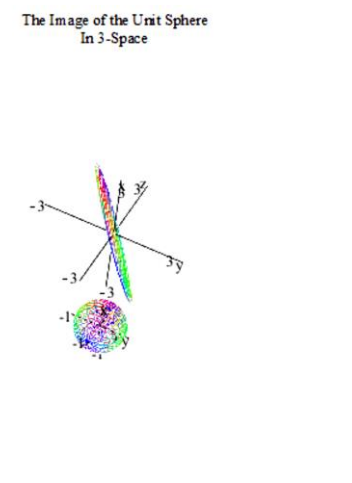
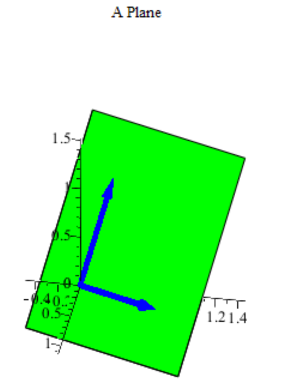


Fig -9(e): The image of the unit sphere under the transformation $T(\vec{x}) = A\vec{x},\ \vec{x} \in \mathbb{R}^3(\mathbb{R})$.

In Tables 9(a), 9(b) and 9(c) below we give the details of the planes including their basis vectors to which the eigenvectors v_1, v_2, v_3 of the correlation matrix A are respectively normal. These three planes are shown respectively in the Figs. 10(a), 10(b) and 10(c). We finish this section by drawing the surface plot of the eigenvectors v_1, v_2, v_3 of the correlation matrix A, which is shown in Fig. 11.

normal vector: <-.5960, .5949, -.5393> equation of plane: -.5960*x+.5949*y-.5393*z = 0. point on plane nearest origin: <0., 0., 0.> basis vectors: <.5949, .7782, .2010>, <-.5393, .2010, .8178> **Table – 9(a):** Details of the plane to which the eigenvector v_1 of A is normal.	 **Fig -10(a)**: Plane to which the eigenvector v_1 of A is normal.

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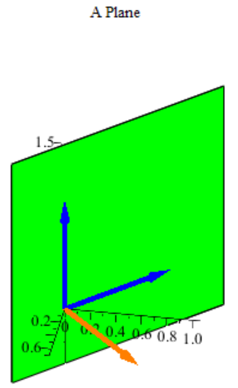
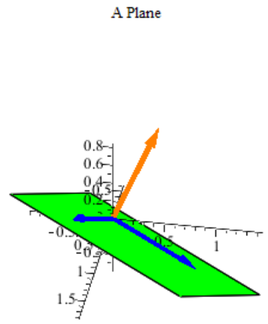
normal vector: <.7104, .7038, -.8637e-2> equation of plane: .7104*x+.7038*y-.8637e-2*z = 0. point on plane nearest origin: <0., 0., 0.> basis vectors: <-.7038, .7104, .3554e-2>, <.8637e-2, .3554e-2, 1.000> **Table – 9(b):** Details of the plane to which the eigenvector v_2 of A is normal.	 **Fig -10(b)**: Plane to which the eigenvector v_2 of A is normal.

normal vector: <-.3744, .3883, .8420> equation of plane: -.3744*x+.3883*y+.8420*z = 0. point on plane nearest origin: <0., 0., 0.> basis vectors: <.3883, .8903, -.2379>, <.8420, -.2379, .4841> **Table – 9(c):** Details of the plane to which the eigenvector v_3 of A is normal.	 **Fig -10(c)**: Plane to which the eigenvector v_3 of A is normal.

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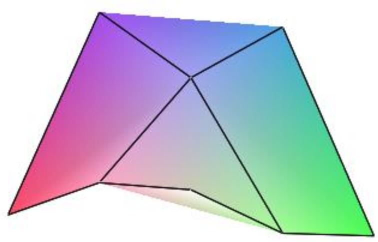

Fig -11: The surface plot of the eigenvectors of the Correlation Matrix A.

5. CONCLUSIONS

In this paper we successfully fitted a model given by (3.1) above to explain the trend of the measured values of SGPT levels of the experimental diabetic rats belonging to the group G4T in the experiments of Ankit Kumar et al. [15] by treating the observed SGPT levels as functions of the two predictors – Day and G4BGm. Based on sorting the various competing models of our numerical experiments for this purpose by the residual sum of squares statistic, the proposed model can explain about 95.71% variation in the levels of SGPT of the rats and thus it poses itself as a reliable mathematical model for the problem under discussion. We also presented our detailed analysis of the correlation matrix of the dataset of Table 1 in section 4 of the paper using various concepts of Linear Algebra and Matrix Theory. We shall also discuss some other competing models for the dataset of Table 1 of this paper in our forthcoming papers. Besides this we shall also delve into the question of studying the variation of the G4SGPT values with G4BGm after removing the parameter b from the model of (3.1) and its various consequences, as we have remarked above. Before parting we would definitely like to disclose here that our studies conducted so far on the variation of the SGPT values with those of G4BGm in Table 1 after the removal of the parameter b from (3.1) have led us to the formulation of a number of closely contesting two-dimensional models which shall also be the subjects of our future papers on this topic.

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